Automation for Separation with CDOs: Dynamic Aircraft Arrival Routes

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Motivation

- Air transportation grows:
  - Beneficial for growing global economy
  - Increased complexity for air traffic controllers (ATCOs)
  - Environmental effects
- Terminal Maneuvering Areas (TMAs) most congested
- Optimization of arrival and departure procedures is needed:
  - Lessen ATCO workload
  - Mitigate environmental impact

Our solution:
- Automatically temporally separated arrivals to reduce complexity and ATCO’s workload
- Aircraft fly according to optimal continuous descent operations (CDOs):
  - Promising solution to mitigate environmental effects, according to ICAO and EUROCONTROL:

  CDOs "allow aircraft to follow a flexible, optimum flight path that delivers major environmental and economic benefits—reduced fuel burn, gaseous emissions, noise and fuel costs—without any adverse effect on safety"
CDOs have shown important environmental benefits w.r.t. conventional (step-down) approaches in TMAs.

Figure source: Performance comparison between TEMO and a typical FMS in presence of CTA and wind uncertainties, by Ramon Dalmau, Xavier Prats, Ronald Verhoeven and Nico de Gelder, DASC 2016.
Previous Work

- **LiU-LFV:**
  - Optimal standard arrival routes (STARs)
  - Time-separated demand-weighted arrival routes (dynamic, for pre-tactical planning), assuming unit edge traversal time
- **UPC:** CDO-enabled optimized arrival procedures (engine-idle, low noise)
- **Here:** Automated time-separated demand-weighted CDO-enabled optimized arrival routes
Grid-based MIP Formulation
Input

- Location and direction of the airport runway
- Locations of the entry points to the TMA
- Aircraft arrival times at the entry points for a fixed time period
- Cruise conditions (altitude, true airspeed, distance to entry point + path distance inside TMA) and aircraft type for CDO profile generation
Optimal arrival tree that:

- Merges traffic from the entries to the runway
- Ensures safe aircraft separation for the given time period

⇒ A set of time-separated \textbf{CDO-enabled} tree-shaped aircraft trajectories optimized w.r.t. the traffic demand during the given period
Operational Requirements

- **No more than two routes merge at a point**: in-degree ≤ 2
- **Merge point separation**: distance threshold L
- **No sharp turns**: angle threshold α, minimum edge length L
- **Temporal separation of all aircraft along the routes**
- **All aircraft fly energy-neutral CDO**: idle thrust, no speed brakes (noise avoidance)
- **Smooth transition** between consecutive trees when switching
Grid-based MIP Formulation

- Square grid in the TMA
- Snap locations of the entry points and the runway into the grid
- Grid cell side of the length L (separation parameter)
Grid-based MIP Formulation

- Square grid in the TMA
- Snap locations of the entry points and the runway into the grid
- Grid cell side of the length l (separation parameter)
- Every node connected to its 8 neighbours
Grid-based MIP Formulation

- Square grid in the TMA
- Snap locations of the entry points and the runway into the grid
- Grid cell side of the length l (separation parameter)
- Every node connected to its 8 neighbours
- Problem formulated as MIP
- Based on flow MIP formulation for Steiner trees
MIP Formulation

VARIABLES

\( x_e \) - decision variable - indicates whether edge \( e \) participates in arrival tree

\( f_e \) - gives the flow on edge \( e = (i, j) \), non-negative

OBJECTIVES

Short flight routes for aircraft

Demand-weighted path length:

\[
\min \sum_{e \in E} \ell_e f_e \quad \Rightarrow \quad \min \beta \sum_{e \in E} \ell_e x_e + (1 - \beta) \sum_{e \in E} \ell_e f_e
\]

Total tree weight:

\[
\min \sum_{e \in E} \ell_e x_e
\]

Arrival tree should “occupy little space”
Constraints

- Flow constraints
- Degree constraints
- Turn angle constraints
- Auxiliary constraints to prevent crossings
- Temporal separation of all aircraft along the routes
- Realistic CDO speed profiles
- Consistency between trees of different time periods
Flow from all entry points reaches runway
Flow of #a/c leaves each entry point
Flow conservation

\[ \sum_{k:(k,i) \in E} f_{ki} - \sum_{j:(i,j) \in E} f_{ij} = \begin{cases} \sum_{k \in \mathcal{EP}} x_k & i = R \\ -x_i & i \in \mathcal{EP} \\ 0 & i \in V \setminus (\mathcal{EP} \cup R) \end{cases} \]  

(1)

\[ x_e \geq \frac{f_e}{|\mathcal{EP}|} \]  

(2)

\[ f_e \geq 0 \]  

(3)

\[ x_e \in \{0, 1\} \]  

(4)

\[ \sum_{k:(k,i) \in E} x_{ki} \leq 2 \]  

(5)

\[ \sum_{j:(i,j) \in E} x_{ij} \leq 1 \]  

(6)

\[ \sum_{k:(k,R) \in E} x_{kR} = 1 \]  

(7)

\[ \sum_{j:(R,j) \in E} x_{Rj} \leq 0 \]  

(8)

\[ \sum_{k:(k,i) \in E} x_{ki} \leq 0 \]  

(9)

\[ \sum_{j:(i,j) \in E} x_{ij} = 1 \]  

(10)

\[ a_e x_e + \sum_{f \in A_e} x_f \leq a_e \]  

(11)

Edges with positive flow are in STAR
Flow non-negative
Degree decision variables are binary

Degree constraints:
Outdegree of every vertex at most 1, maximum indegree is 2.
Runway only one ingoing, entry points only one outgoing edge.

If an edge \( x_e \) the angle to the consecutive segment of a route is never smaller than \( \alpha \)
Constraints

Auxiliary Constraints to Prevent Crossings

Why? Temporal Separation may enforce paths that are not shortest, hence, crossings may appear

For all points except last column, last row, entries and rwy:
\[
\sum_{j=1}^{n} x_{i,j+1} + x_{i+1,j} + x_{i+n,j+1} + x_{i+1,j+n} \leq 1
\]
\[
\forall i \in V' \setminus \{P \cup r\} : i + 1 + n, i + n, i + 1 \not\in P \cup r
\]
\[
V' = V \setminus \{\text{last row}\} \setminus \{\text{last column}\}
\]

For different entry point locations:
\[
\sum_{j=1}^{n} x_{i,j+1} + x_{i+n,j+1} + x_{i+1,j+n} \leq 1 \forall i \in P
\]
\[
\sum_{j=1}^{n} x_{i,j+1} + x_{i+1,j+n} \leq 1 \forall i : i + 1 \in P
\]
\[
\sum_{j=1}^{n} x_{i,j+1} + x_{i+n,j+1} + x_{i+1,j+n+1} \leq 1 \forall i : i + n \in P
\]
\[
\sum_{j=1}^{n} x_{i,j+1} + x_{i+n,j+1} + x_{i+1,j+n+1} \leq 1 \forall i : i + n + 1 \in P
\]

Constraints

Temporal Aircraft Separation

Assumption: unit time $u$ to cover a single edge

More variables: $y_{a,j,t}$ - binary, shows a/c $a$ at node $j$ at time $t$

$x_{e,b}$ - binary: edge $e$ in the route from entry point $b$

Connect to $x_e$

$$x_{e,b} \leq x_e \forall b \in P, \forall e \in E$$

plus several other constraints

Set:

$$y_{a,b,t} = \begin{cases} 1 & \forall b \in P, \forall a \in A_b \\ 0 & \forall b \in P, \forall a \in A \setminus A_b, \forall t \in T \end{cases}$$

$$y_{a,b,t} = \begin{cases} 0 & \forall b \in P, \forall a \in A_b, \forall t \in T \setminus \{t^b_a\} \\ 1 & \forall b \in P, \forall a \in A \setminus A_b, \forall t \in T \setminus \{t^b_a\} \\ \sum_{(k,j) \in E} x_{(k,j)} \forall b \in P, \forall a \in A, \forall j \in V \setminus P, \forall t \in T \end{cases}$$

Forward the information on the times at which $a$ arrives at nodes along the route from $b$ to the rwy

$$\sum_{j:(j,k) \in E} x_{(j,k),b} \times y_{a,j,t} = y_{a,k,t+u} \forall b \in P, \forall a \in A_b, \forall k \in V \setminus P, \forall t \in \{0, \ldots, T - u\}$$

Temporal separation:

$$\sum_{\tau = t}^{t+\sigma-1} \sum_{a \in A} y_{a,j,\tau} \leq 1 \forall j \in V, \forall t \in \{0, \ldots, T - \sigma + 1\}$$

$\sigma$ - separation parameter

Not linear $\implies$ we linearise using a new variable $z_{a,j,k,b,t}$

$$z_{a,j,k,b,t} - y_{a,k,t+u} = 0 \forall b \in P, \forall a \in A_b, \forall k \in V \setminus P, \forall t \in \{0, \ldots, T - u\}$$
Constraints

- Flow constraints
- Degree constraints
- Turn angle constraints
- Auxiliary constraints to prevent crossings
- Temporal separation of all aircraft along the routes
- Realistic CDO speed profiles
- Consistency between trees of different time periods
• The state vector $x$ represents the fixed initial conditions of the aircraft: TAS $v$, altitude $h$ and distance to go $s$

• To achieve environmentally friendly trajectories, idle thrust is assumed and speed-brakes use is not allowed throughout the descent $\rightarrow$ energy-neutral CDO

• The flight path angle is the only control variable in this problem $\rightarrow$ control vector $u$

\[
x = [v, h, s]
\]

\[
u = [\gamma]
\]
A point-mass representation of the aircraft reduced to a “gamma-command” is considered, where vertical equilibrium is assumed → **Dynamic constraints** $f$

Path constraints $h$ are enforced to ensure that the aircraft airspeed remains within operational limits, and that the maximum and minimum descent gradients are not exceeded

Terminal constraints $\psi$ fix the final states vector

**Dynamic constraints**

$$f = \begin{bmatrix} \dot{v} \\ \dot{h} \\ \dot{s} \end{bmatrix} = \begin{bmatrix} \frac{T_{idle}-D}{m} - g\gamma \\ v\gamma \\ v + w \end{bmatrix}$$

**Path constraints**

$$h = \begin{bmatrix} v_{CAS,min} - v_{CAS} \\ v_{CAS} - VMO \\ M - MMO \\ \gamma \\ \gamma_{min} - \gamma \end{bmatrix} \leq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

**Terminal constraints**

$$\psi = \begin{bmatrix} v - v_f \\ h - h_f \\ s - s_f \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$
Realistic CDO Speed Profiles

- The trajectory is divided in two phases: the latter part of the cruise phase prior the top of descent (TOD) and the idle descent
- The original cruise speed is not modified after the optimization process, so the two-phases optimal control problem can be converted into a single-phase optimal control problem
- BADA V4 is used to model the aircraft performance

\[ J = \frac{f}{v_{cruise}} + \int_{t_0}^{t_f} (f_{idle} + Cl) \, dt \]

Constraints

- Flow constraints
- Degree constraints
- Turn angle constraints
- Auxiliary constraints to prevent crossings
- Temporal separation of all aircraft along the routes
- Realistic CDO speed profiles
- Consistency between trees of different time periods
Integration of CDO-enabled Realistic Speed Profiles

Substitute: $y_{a,j,t}$ with $y_{a,j,p,n,t}$ - binary, indicates whether a/c $a$ using speed profile $p$ occupies the $n$-th vertex $j$ at time $t$.

Substitute the corresponding equations with:

\[ \sum_{p \in S(a)} y_{a,b,p,1,t}^{a} = 1 \quad \forall b \in P, \forall a \in A_b \]
\[ y_{a,b,p,k,t}^{a} = 0 \quad \forall b \in P, \forall a \in A_b, \forall p \in S(a), \forall k \neq 1 \in \mathcal{L} \]
\[ y_{a,b,1,t} = 0 \quad \forall b \in P, \forall a \in A_b, \forall p \in S(a), \forall t \in T \setminus \{t_{a}^{b}\} \]
\[ y_{a,b,p,k,t} = 0 \quad \forall b' \neq b \in P, \forall a \in A_b, \forall p \in S(a), \forall k \in \mathcal{L}, \forall t \in T \]
\[ y_{a,b,1,t} = 0 \quad \forall b \in P, \forall a' \neq a \in A_b, \forall p \in S(a), \forall t \in T \]
\[ y_{a,j,p,k,t} \leq \sum_{i \in V \setminus \{(i,j)\} \in E} x_{(i,j)} \quad \forall j \in V \setminus P, \forall a \in A, \forall p \in S(a) \]

Compute $\ell(b)$ - path length from $b$ to the rwy
\[ \ell(b) = \sum_{(i,j) \in \mathcal{E}} x_{(i,j),b} \]

For each a/c $a$ arriving from $b$ we pick the speed profile from $S(a)$ that has the length $\ell(b)$, i.e., we want:
\[ y_{a,b,\ell(b),1,t}^{a} = 1 \quad \text{and} \quad y_{a,b,p,1,t}^{a} = 0 \quad \forall p \neq \ell(b) \]

$\ell(b)$ is a variable $\implies$ We use auxiliary binary variables and constraints to achieve this.

Separation constraint:
\[ \sum_{\tau=t}^{t+\sigma-1} \sum_{a \in A} \sum_{p \in S(a)} \sum_{k \in \mathcal{L}} y_{a,j,p,k,\tau} \leq 1 \quad \forall j \in V, \forall t \in \{0, \ldots, T - \sigma + 1\} \]

$\sigma$ - separation parameter
Constraints

Consistency between trees of consecutive time periods

Define: \( x_{ij} \) and \( x_{ij}^{old} \) - edge indicators for current and previous periods

\( U \) - limits the number of differing edges in the two trees

\[
\begin{align*}
ax_{ij} & \leq x_{ij} - x_{ij}^{old} \quad \forall (j, i) \in E \\
ax_{ij} & \leq x_{ij}^{old} - x_{ij} \quad \forall (j, i) \in E \\
\sum_{(i, j) \in E} ax_{ij} & \leq U
\end{align*}
\]
Experimental Study: Stockholm Arlanda Airport
Data: Stockholm Arlanda airport arrivals during one hour of operation

Source: EUROCONTROL DDR2, BADA 4

High-traffic scenario on October 3, 2017, time: 15:00 - 16:00

Solved using GUROBI

Run on a powerful Tetralith server, provided by SNIC, LIU: Intel HNS2600BPB nodes with 32 CPU cores and 384 GiB RAM
• Cruise conditions are obtained from DDR2
• TOD position and descent phase are optimized
• Same time at the entry point for different path lengths inside TMA
CDO profiles inside TMA

- A set of realistic alternative speed profiles for different possible route lengths inside TMA
- Generated for all a/c types arriving to Arlanda during the given period
- Used as input to MIP

Example of A320 speed profiles for different path lengths inside TMA
Results: Stockholm Arlanda Airport

**Tree 1:** time: 15:00 - 15:30 (10 a/c)

**Tree 2:** time: 15:30 - 16:00 (7 a/c)
Results: Stockholm Arlanda Airport

- Tree 1: time: 15:00 - 15:30 (10 a/c)
- Tree 2: time: 15:30 - 16:00 (7 a/c)
- Optimized for 30 min intervals (longer periods may be sub-optimal. Note: time within TMA 5-18 min)
- U = 23 provides consistency between the trees
- Separation: 2 min, ~6 nm
- 17 out of 22 arrivals scheduled
- 5 filtered out, because of:
  - Initial violation of separation at entry points
  - Potential overtaking problem
  - In general, about 10-15% are not scheduled
Comparison against historical trajectories (Open Sky Network)
## Time Schedule

<table>
<thead>
<tr>
<th>Arrivals</th>
<th>Entry point</th>
<th>Simulated time [min]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Entry</td>
</tr>
<tr>
<td>a1</td>
<td>Ent1 (North)</td>
<td>3</td>
</tr>
<tr>
<td>a2</td>
<td>Ent2 (West)</td>
<td>8</td>
</tr>
<tr>
<td>a3</td>
<td>Ent3 (East)</td>
<td>13</td>
</tr>
<tr>
<td>a4</td>
<td>Ent4 (South)</td>
<td>4</td>
</tr>
<tr>
<td>a5</td>
<td>Ent4</td>
<td>18</td>
</tr>
<tr>
<td>a6</td>
<td>Ent2</td>
<td>17</td>
</tr>
<tr>
<td>a7</td>
<td>Ent1</td>
<td>17</td>
</tr>
<tr>
<td>a8</td>
<td>Ent1</td>
<td>21</td>
</tr>
<tr>
<td>a9</td>
<td>Ent2</td>
<td>19</td>
</tr>
<tr>
<td>a10</td>
<td>Ent3</td>
<td>28</td>
</tr>
<tr>
<td>a11</td>
<td>Ent4</td>
<td>34</td>
</tr>
<tr>
<td>a12</td>
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<tr>
<td>a13</td>
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<td>a14</td>
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<tr>
<td>a15</td>
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<tr>
<td>a16</td>
<td>Ent4</td>
<td>53</td>
</tr>
<tr>
<td>a17</td>
<td>Ent2</td>
<td>57</td>
</tr>
</tbody>
</table>
$t = 15:00$
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t = 15:05
$t = 15:07$
t = 15:08
t = 15:09
t = 15:10
$t = 15:11$
t = 15:12
$t = 15:13$
t = 15:14
t = 15:15
t = 15:16
$t = 15:17$
t = 15:18
t = 15:19
$t = 15:20$
t = 15:21
t = 15:22
t = 15:23
t = 15:24
t = 15:25
t = 15:26
t = 15:27
t = 15:28
t = 15:29
$t = 15:30$
$t = 15:30$
t = 15:30
$t = 15:30$
$t = 15:31$
t = 15:32
Conclusions and Future Work
Conclusions and Future Work

Conclusions

- Flexible optimization framework for dynamic route planning inside TMA
- Automated spatial and temporal separation
- Environmentally-friendly speed profiles (CDO)
- Applicable to any other realistic speed profiles
- May be used for TMA capacity evaluation
- Account for uncertainties due to variations in arrival times
- Solve overtaking problem (allow non-optimal profiles, or route stretching)
- Consider fleet diversity
- Elaborate on implementation possibilities, link to the future operational enablers (data links, technologies) for air-ground synchronisation (EPP)
THANKS.