Entropy solutions and an inverse problem of a scalar conservation law modelling sedimentation in vessels with varying cross sectional area

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Abstract

The sedimentation of an ideal suspension in a vessel with variable cross-sectional area can be described by an initial-boundary value problem for a scalar nonlinear hyperbolic conservation law with a nonconvex flux function and a weight function that depends on spatial position. The sought unknown is the local solids volume fraction. For the most important cases of vessels with downward-decreasing cross-sectional area and flux function with at most one inflection point, entropy solutions of this problem are constructed by the method of characteristics. Solutions exhibit discontinuities that mostly travel at variable speed, i.e., they are curved in the space-time plane. These trajectories are given by ordinary differential equations that arise from the jump condition. It is shown that three qualitatively different solutions may occur in dependence of the initial concentration. A comparison of the solution obtained by using the method of characteristics with the numerical solution using an approximation of the flow function given by the *Godunov method* is also presented. In addition it is displayed the solution of an inverse problem for flux identification using the entropy solution and interpolation techniques via cubic spline. Related models also arise in flows of vehicular traffic, pedestrians, and in pipes with varying cross-sectional area.

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