Automatic Design of Aircraft Arrival Routes with Limited Turning Angle

Tobias Andersson Granberg, Tatiana Polishchuk, Valentin Polishchuk, Christiane Schmidt
Introduction: Air transportation, SIDs + STARs

Grid-based IP formulation

Experimental Study: Arlanda Airport

Conclusion/Outlook
Air transportation:
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    ○ avoid creating conflict points
At most airports predesigned standard routes for departure and arrival:
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Standard Instrument Departures (SIDs) and Standard Terminal Arrival Routes (STARs)
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**Standard Instrument Departures (SIDs)** and **Standard Terminal Arrival Routes (STARs)**

**STAR**
Stockholm, RWY 01L/01R

**SID**
Stockholm, RWY 01L
SIDs/STARs:
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- Designed manually
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• No optimal routes for any specific criteria
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- Here: mathematical programming framework for finding optimal STAR merge trees
Optimal STAR merge trees

Input:
Optimal STAR merge trees

Input:
locations of the entry points to the TMA
Optimal STAR merge trees

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locations of the entry points to the TMA
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1. **No more than two routes merge at a point:** in-degree ≤ 2
Optimal STAR merge trees

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location and direction of the airport runway

Output:
arrival tree that merges traffic from the entries to the runway, i.e., a tree that has the entries as leaves and the runway as the root (arborescence oriented differently than usual)

1. **No more than two routes merge at a point:** in-degree \( \leq 2 \)
2. **Merge point separation:** distance threshold \( L \)
Input:
locations of the entry points to the TMA
location and direction of the airport runway

Output:
arrival tree that merges traffic from the entries to the runway,
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2. **Merge point separation:** distance threshold $L$
3. **No sharp turns:** angle threshold $\alpha$, minimum edge length $L$
Optimal STAR merge trees

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5. **STAR–SID separation:**
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   STAR–SID crossings far from the runway,
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2. **Merge point separation:** distance threshold $L$
3. **No sharp turns:** angle threshold $\alpha$, minimum edge length $L$
4. **Obstacle avoidance**
5. **STAR–SID separation:**
   STAR–SID crossings far from the runway,
   where arriving and departing planes sufficiently separated
   vertically (difference of descend and climb slopes)
Objective function:
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- Short flight routes for aircraft
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Pareto frontier of multicriteria optimization problem:
set of Pareto optimal solutions (cannot be improved with respect to one of the objectives without sacrificing on the other)
Grid-based IP formulation
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- Square grid in the TMA
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- Side of the grid pixel: \( L \) (merge point separation)
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- G = (V,E):
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- Side of the grid pixel: L (merge point separation)

- G = (V,E):
  - Every grid node connected to its 8 neighbors
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- Side of the grid pixel: L \( \Rightarrow \) merge point separation

- \( G = (V,E) \):
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  - \( G \) is bi-directed
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  - Only exceptions:
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- \( l_{i,j} \) length of an edge \( (i, j) \)
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- IP formulation is based on flow IP formulation for Steiner trees (Min Cost Flow Steiner arborescence)
Grid-based IP formulation
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\( x_e \) decision variables: edge \( e \) participates in the STAR.
Grid-based IP formulation

\[ x_e \text{ decision variables: edge } e \text{ participates in the STAR.} \]
\[ f_e \text{ flow variables: gives the flow on edge } e = (i, j) \text{ (i.e., from } i \text{ to } j \text{).} \]
Grid-based IP formulation

- $x_e$: decision variables: edge $e$ participates in the STAR.
- $f_e$: flow variables: gives the flow on edge $e = (i, j)$ (i.e., from $i$ to $j$)

$$
\sum_{k: (k,i) \in E} f_{ki} - \sum_{j: (i,j) \in E} f_{ij} = \begin{cases}
|\mathcal{P}| & i = r \\
-1 & i \in \mathcal{P} \\
0 & i \in V \setminus \{\mathcal{P} \cup r\}
\end{cases}
$$

- $x_e \geq \frac{f_e}{N}$ for all $e \in E$
- $f_e \geq 0$ for all $e \in E$
- $x_e \in \{0, 1\}$ for all $e \in E$
Objective functions:

\[
\begin{align*}
\text{min } & \sum_{e \in E} \ell_e f_e \\
\text{min } & \sum_{e \in E} \ell_e x_e
\end{align*}
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Objective functions:

\[
\min \sum_{e \in E} \ell_e f_e \quad \text{(1)}
\]

\[
\min \sum_{e \in E} \ell_e x_e \quad \text{paths length}
\]

\[
\min \sum_{e \in E} \ell_e x_e \quad \text{tree weight}
\]

\[
\min \sum_{e \in E} \ell_e x_e \quad \text{(2)}
\]
Grid-based IP formulation

Degree constraints:

$$\sum_{k:(k,i)\in E} x_{ki} \leq 2 \quad \forall i \in V \setminus \{P \cup r\}$$

$$\sum_{j:(i,j)\in E} x_{ij} \leq 1 \quad \forall i \in V \setminus \{P \cup r\}$$

$$\sum_{k:(k,r)\in E} x_{kr} = 1$$

$$\sum_{j:(r,j)\in E} x_{rj} \leq 0$$

$$\sum_{k:(k,i)\in E} x_{ki} \leq 0 \quad \forall i \in P$$

$$\sum_{j:(i,j)\in E} x_{ij} = 1 \quad \forall i \in P$$
Turn angle constraints:
Grid-based IP formulation

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\[ A_e \]
Grid-based IP formulation

Turn angle constraints:

\[ a_e = |A_e| \]
Grid-based IP formulation

Turn angle constraints:

\[ a_e = |A_e| \]

\[ a_e x_e + \sum_{f \in A_e} x_f \leq a_e \quad \forall e \in E \]
SID constraints:
We disallow STAR edges to intersect SID edges within distance $d$ from the runway.
Experimental Study: Arlanda Airport
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Stockholm TMA:
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- Arlanda’s runway 19L
Experimental Study: Arlanda Airport

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- Four main entry points: NILUG, XILAN, HMR, and ARS
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Pareto frontier:
Experimental Study: Arlanda Airport

Pareto frontier:

Pareto optimal solutions:
Obstacle avoidance:
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Increased Number of Entry Points:
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paths length
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Experimental Study: Arlanda Airport

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**Diagram:**

- Paths length
- Tree weight
Increased Number of Entry Points:

paths length

tree weight
Experimental Study: Arlanda Airport

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Increased Number of Entry Points:

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serve the airlines’ request for short trajectories best
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quite dense network of routes ➔
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Experimental Study: Arlanda Airport

Increased Number of Entry Points:

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Quite dense network of routes

Hard to control the traffic

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Helpful to use linear combination of these two functions
Experimental Study: Arlanda Airport

Increased Number of Entry Points:

Paths length

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Increased Number of Entry Points:

- **Paths Length**
- **Tree Weight**

Solutions for large number of entry points could be used to suggest the number and location of entry points for a design from scratch.

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Experimental Study: Arlanda Airport

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2 entry points
Increased Number of Entry Points:

- **Paths Length**

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Experimental Study: Arlanda Airport

SID constraints:

Each tree:
within approximately
2 CPU hours
(105 B&B nodes)
SID constraints:
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Experimental Study: Arlanda Airport

SID constraints:

![Graph showing SID constraints with paths length and radius on a grid.](image)
Conclusion/Outlook
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Easily integrates constraints from the departure routes
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Static obstacles, e.g., no-fly zones, can be added
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Simultaneous design of both SIDs and STARs
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3D routes