DAA1, L1 (I)

Proof Lemma 1.60:

(a) "⇒": If there is a set $X \subset V(G)$ with $r \in X$, $v \in V(G) \setminus X$ and $\delta(X) = \emptyset$, there can be no r-v-path, so G is not connected.

"⇐": If G is not connected, there is no r-v-path for some r and v. Let R be the set of vertices reachable from r. We have $r \in R$, $v \notin R$ and $\delta(R) = \emptyset$.

(b) analogously

□

Proof Lemma 1.63

Let $e = (x, y)$. We label the vertices of G by the following procedure:

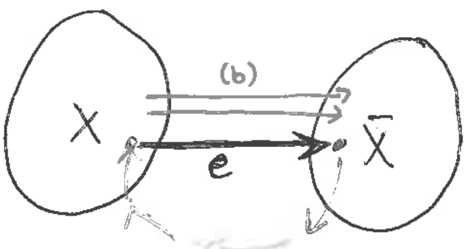
First label y.
In case v is already labelled and w is not, we label w if there is a black edge (v,w), a red edge (v,w) or a red edge (w,v).
In this case, we write $\mathrm{pred}(w) := v$.

When the labelling procedure stops, there are two possibilities

(1): x has been labelled. Then the vertices $x, \mathrm{pred}(x), \mathrm{pred}(\mathrm{pred}(x)), \ldots, y$ form an undirected circuit with the properties (a).

(2): x has not been labelled. Then let R consists of all labelled vertices. Obviously, the undirected cut $\delta^+(R) \cup \delta^-(R)$ has the properties (b).

Suppose that an undirected circuit C as in (a) and an undirected cut $\delta^+(X) \cup \delta^-(X)$ as in (b) both exist. All edges in their (nonempty) intersection are black, they all have the same orientation w.r.t. to C, and they all leave X or all enter X ↯ contradiction



Examples: Θ, O, Ω

Bsp.: ◦ $2n^2 - 1 \in \Theta(n^2)$

$c_1 = 1$ $\quad 0 \leq c_1 \cdot n^2 \leq 2n^2 - 1 \leq 2n^2 \quad \forall n \geq n_0 = 1$

$c_2 = 2$ $\quad 0 \leq 1 n^2 \leq \cancel{2}n^2 + 1 - 1 \leq 2n^2 - 1 \leq 2n^2$

$n_0 = 1$

◦ $2n^2 - 1 \in O(n^2)$

$\in O(n^3) \quad c_2 = 2, \; n_0 = 1$

$0 \leq 2n^2 - 1 \leq 2n^3 \quad \forall n \geq n_0 = 1$

• $n \log n \in O(n^2)$

$c_2 = 1$ $\quad 0 \leq n \cdot \log n \leq n^2 \quad \forall n \geq n_0$

$n_0 = 1$

• $3n^3 + 4 \in \Omega(n^2)$

$c = 3$ $\quad 0 \leq 3 \cdot n^2 \leq 3n^3 \leq 3n^3 + 4 \quad \forall n \geq n_0 = 1$

$n_0 = 1$