A2) Bottleneck STs

To show: $\underbrace{T \text{ MST in } G}_{A} \Rightarrow \underbrace{T \text{ BST in } G}_{B}$ $\neg B \Rightarrow \neg A$

We assume, T is not a BST in an undirected graph G.
We show: T is not an MST.
Let e be the edge of T with highest weight.
$\Rightarrow \exists$ two subtrees T_1, T_2 such that e connects these two.
T is not a BST $\Rightarrow \exists$ edge e' with $c(e') < c(e)$ that also connects T_1 and T_2
Let T' be the tree $T' = T_1 \cup T_2 \cup \{e'\}$.
Let $S(X)$ be the sum of the edge weights in a tree X.
$\Rightarrow S(T) - S(T') = c(e) - c(e') > 0$ $\lightning$ T is MST in G

A3) Planar MSTs

Let T be a tree that contains $\{u,v\}$.
Let w be another point in the lune of u and v.
$\Rightarrow c(\{u,w\}) < c(\{u,v\})$
$c(\{w,v\}) < c(\{u,v\})$
Deleting $\{u,v\}$ from T splits it in 2 CC: U (with u) and V (with

- w in $U \Rightarrow T - \{u,v\} + \{v,w\}$
- w in $V \Rightarrow T - \{u,v\} + \{u,w\}$

} both have lower total weight

$\Rightarrow$ no point within the lune of u and v.

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