Conference "State of-the-arts in the classical separability theory for differential equations" January 6 11, 2004
Department of Mathematics, Linköping University, Sweden

Alphabetical list of talks

Vladislav Bagrov

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Title: Separation of variables and classification of Stäckel spaces in gravitation theory.

Abstract:

Sergio Benenti

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Title: Torsionless conformal Killing tensors and orthogonal separation.

Abstract: Basic properties of torsionless conformal Killing tensors and their relations with the separation of variables are given.

Maciej Blaszak

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Title: Bi-presymplectic representation of separable systems

Abstract: A bi-presymplectic representation of bi-Poisson one-Casimir chains for Liouville integrable systems is constructed. Its relation with the geometric separability theory is shown.

Ljudmila Bordag

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Title: Quasi periodic vortex structures in two-dimensional flows in an inviscid incompressible fluid.

Abstract: We investigate a two-dimensional case of a steady motion of an inviscid incompressible fluid. The most general form for an equation of the motion in this case is $\Delta u(x,y) = F(u(x,y))$ where $u(x,y)$ is a stream function of local coordinates x,y, Δ is the Laplace operator and F(.) an arbitrary function. The function F is the measure of the flow vorticity. At present it seems that the only way to consider an equation of the type with non trivial function F with analytical methods is to use the inverse scattering method. To the list of integrable equations

of the form belong the Sine - Laplace equation (SL), the Cos - Laplace equation, the Sinh -Laplace equation (SHL), the Liouville equation (an explicit integrable one), the Tzitzeica equation and the Cosh - Laplace equation. The first five of these equations allow a Lamb - substitution, which was used to describe the simplest types of a fluid motion. The characteristics of corresponding flows evince typical properties connected with the separation of variables in the equations. In terms of flow properties we can describe the flows in corner-like regions or in channels with straight lines boundaries.

The idea to look on the solutions of the Cosh - Laplace equation (CHL) $\Delta u(x,y)=\pm\cosh(u(x,y))$ came from this point of view. This equation doesn't admit the Lamb substitution, hence we can get a more interesting and complicated structure of singularities. It is one of the equations which can direct us to a description of flows in regions with non strait line boundaries. The special technique of the finite-gap integration allows to get real solutions of the CHL equation using a compact Riemann surface with a proper symmetry.

We study the first non trivial case and take a Riemann surface of the genus g=3. The hydrodynamical interpretations of the finite-gape solutions is non trivial and we try to reach a complete understanding of the described processes in fluid. To reach this goal we will take a Riemann surface with additional symmetry properties.

We present the four five parametric families of exact solutions. These solutions are given in term of Jacobian elliptic functions, which allows direct investigations of all physical properties. We were also able to found the explicit formulas for the singularities curves. From point of view of the algebraic geometry it is interesting to remark that it was possible to describe the complete structure of the real part of theta divisor.

Francesco Calogero

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Title: Isochronous systems.

Abstract: Isochronous systems are characterized by the existence of many isochronous solutions -- namely solutions completely periodic with a fixed period (independent of the initial data). Almost any dynamical system can be modified so that it becomes isochronous, in particular -- in the context of the initial-value problem -- so that it possesses isochronous solutions emerging out of an open set of initial data, having the full dimensionality of the relevant phase space. In this sense isochronous systems are not rare. And in some sense they are superintegrable -- although this notion can be made precise only for Hamiltonian systems with a finite number of degrees of freedom, whereas the results mentioned above also apply to non Hamiltonian systems, and also to systems with an infinite number of degrees of freedom (nonlinear evolution PDEs). The simple idea underlining this development will be explained, and many interesting examples will be exhibited -- mainly, but not exlusively, many-body problems.

Claudia Chanu

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Title: Conformal Killing tensors and separation of variables for the Hamilton-Jacobi equation with fixed value of the energy.

Abstract: We provide a suitable definition of separation of variables for a null Hamilton-Jacobi equation and deduce several equivalent conditions generalizing those of Levi-Civita. By considering the particular case of a natural Hamiltonian, we characterize its separation through conformal Killing tensors satisfying suitable properties, recovering the classical results and giving some new characterizations.

Gregorio Falqui
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Title: Gaudin Models and Bending Flows: a geometrical set-up

Abstract: In this talk I will briefly discuss the formulation of classical
Gaudin models of sl(r) ``spin-spin" interaction both in the Lax setting
and in the framework of bihamiltonian geometry.
In particular, I will focus on the so-called homogeneous model, in connection
with the so--called "bending flows" in the moduli space of polygons in
Euclidean space introduced by Kapovich and Millson.
The problems of complete integrability and separation of variables will be
discussed.

Ernie Kalnins
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Title: Separation of variables for spaces of constant curvature.

Abstract: In this talk the classification of all separable systems for Remannian spaces of constant curvature is discussed.The complete solution to this problem in the case of real Riemannian spaces of constant curvature is contrasted with the case of complex spaces where the solution is not known. The problems to be overcome in the complex case are outlined as well as some ideas that may resolve them. The talk is well equipped with examples explaining the nature of the difficulties.

Igor Komarov
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Title: Some features of separation of variables in classical and quantum integrable systems

Abstract: SoV and Liouville theorem. Commutativity of the SV. Definition by Sklyanin in classical and quantum mechanics. Non uniqueness of SoV Numerical advantages of the SoV in spectral problems. SoV as a case studies. C.Jacobi idea: Direct and inverse problem of the SoV. Spational SoV - curvilinear coordinates Laplace operators and SoV. Hamiltonians quadratic form in coordinates and momenta Lax systems: separated coordinates as poles of the Baker-Akhiezer function. Influence of its normalization Baxter operator and Baxter equation. Relation with B\"aklund transform. Baxter equation and SoV. Bethe Ansatz and the SoV. Kernel of unitary transformation from initial to separated representation. Example: Kowalevski top

Edwin Langmann
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Title: On the elliptic (quantum) Calogero-Sutherland model

Abstract: I discuss an algorithm to solve the elliptic Calogero-Sutherland model and its relation to separability.

Hans Lundmark
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Title: Driven Newton equations and separable time-dependent potentials

Abstract: I will describe a class of time-dependent potentals that are separable in a sense which generalizes classical Stäckel separability. These potentials are closely connected to the theory of driven Newton equations of cofactor type.

Franco Magri
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Title: The Kowalewski method.

Abstract: Consider an integrable system with two degres of freedom (to start with) of which you know two integrals of motion. An example is the Kowalevski top. Suppose I give you a second –order polynomial and I ask you the question: are the zeros of this polynomial the separation variables for the given integrable system? I propose an intrinsic criterion to answer this question (which does not require to compute explicitjy the zeroes of the polynomial), and I show how to use this criterion to rediscover (in a way completely different from Kowalevski) the separation varaibles for the Kowalevski top.

Krzysztof Marciniak
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Title: Separation of variables for triangular cofactor systems.

Abstract: In this talk I will investigate the so called triangular cofactor systems, that is dynamical systems of the form $\ddot{q}=M(q)$ with the 'force' M(q) of a triangular ('completely driven') form and with one 'essential' quadratic in momenta integral of motion. I will demonstrate a procedure of variable separation for these systems that allows for solving these systems in quadratures. This procedure is not related to the standard Hamilton-Jacobi separation theory. I will also present some results on classification of separation coordinates for these systems in two and three dimensions. This is a joint work with H. Lundmark and S. Rauch-Wojciechowski.

Willard Miller Jr.
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Title: Variable separation in mathematical physics: From intuitive concept to computational tool (joint with E. G. Kalnins).

Abstract: We review the basic intuitive definition of separation of variables (additive, multiplicative and functional) for the partial differential equations of mathematical physics and show how it leads to constructive methods for finding separable systems and their solutions. In many important cases there are deep connections with Lie theory and integrable systems, but there are interesting applications where no such link seems to exist. We survey the basic results relating orthogonal separation and R-separation for Hamilton-Jacobi equations and Schroedinger eigenvalue equations on n-dimensional Riemannian manifolds to the symmetries of these equations, and we present methods for classifying the possible separable coordinate systems. We conclude with a survey of results and challenging problems involving superintegrable systems, typically multiseparable systems in which the eigenvalues and eigenfunctions of the various separable systems can be determined by algebraic methods.

Marco Pedroni
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Title: Separability and Stäckel separability: a bi-Hamiltonian viewpoint.

Abstract: From the point of view of bi-Hamiltonian geometry, a separable system is described by a bi-Lagrangian (integrable) foliation on a nondegenerate bi-Hamiltonian manifold. Given such a foliation and a set of independent functions which generate the foliation, one can look for conditions (on the functions) ensuring that they appear linearly in the separation relations. More generally, one can try to geometrically characterize the bi-Lagrangian distributions admitting a set of generators with the above-mentioned property.

Manuel Ranada
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Title: Separable potentials in spaces of constant curvature.

Abstract: The properties of the potentials defined on the two-dimensional spaces of constant curvature (sphere, Euclidean plane, and hyperbolic plane) are studied.
First the form of the Hamiltonians and the separability of the Hamilton-Jacobi equation is considered, and then the characteristics of the associated constants of motion are analyzed. This approach makes use of the curvature κ as a parameter, in such a way that,particularizing for $\kappa>0$, $\kappa=0$, or $\kappa<0$, the corresponding properties are obtained for the system (or the equations) on the sphere S^2, the Euclidean plane E^2, or the hyperbolic plane H^2, respectively.

Giovanni Rastelli
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Title: From eigenvalues of Killing tensors to separable coordinates in pseudoRiemannian manifolds.

Abstract: We illustrate an algebraic method based on the eigenvalues of suitable Killing tensors which provide to compute the equations of the foliations forming a separable web.

Stefan Rauch-Wojciechowski
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Title: Solution of the Jacobi problem of separation of variables and theory of quasipotential Newton equation.

Abstract: Our solution of the Jacobi problem of finding separation variables for natural Hamiltonian systems H=(1/2)p^2+V(q) is explained. It has a form of an effective criterion that for any given potential V(q) tells whether there exist suitable separation coordinates x(q) and how to find these coordinates, so that the Hamilton-Jacobi equation of the transformed Hamiltonian is separable. The main reason for existence of such criterion is the fact that for separable potentials V(q) all integrals of motion depend quadratically on momenta and that all orthogonal separation coordinates stem from the generalized elliptic coordinates. This criterion is directly applicable to the problem of separating multidimensional stationary Schrödinger equation of quantum mechanics.
Theory of quasipotential Newton equations treats equations of the form (d^2q/dt^2)=M(q) that admit n quadratic integrals of motion. It naturally generalizes theory of separable potential systems (d^2q/dt^2)= -gradV(q). We show that under certain technical assumptions such Newton equations admit a Hamilton-Poisson structure in an extended 2n+1 dimensional phase space and are integrable by embedding into a Liouville integrable system. There are two characterizations of these systems: one through a Poisson pencil and another one through a set of Fundamental Equations. For a generic qusipotential Newton equations separation variables are given and such systems are shown to be equivalent to a Stäckel separable Hamiltonian system. This theory is illustrated by examples of driven and triangular Newton equations.

Kjell Rosquist
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Title: The classical R-matrix from a geometric point of view.

Abstract: A geometric formulation of the Lax pair equation is described. In this picture, the integrable dynamical system is realized as a geodesic flow on a curved space which carries all the dynamical information in its metric, or equivalently, in its geodesic Hamiltonian. The geometric version of the Lax pair equation then appears as a tensorial equation written entirely in terms of configuration space tensors, one of which may be viewed as a generalized third rank Killing tensor. A similar formulation of the classical R-matrix is also given. The specific geometric nature of the classical R-matrix provides a natural explanation of its transformation properties.

Boris Samsonov
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Title: Separation of variables in the Laplace-Beltrami equation in a flat space.

Abstract: It is proven that all orthogonal coordinate systems admitting separation of variables in this equation are exhausted by cyclide coordinates or their degenerate forms.
For a particular case of the D'Alambert equation the problem of identities in the enveloping algebra of the symmetry algebra is discussed. Solutions of the free particle Schroedinger equation in separated variables are analyzed from the point of view of the D'Alambert equation.

Mariano Santander
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Title: Separability in Spaces of constant curvature: II Some Geometrical problems related with Separability on the two-dimensional Sphere and the Hyperbolic plane.

Abstract:

Willy Sarlet
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Title: Bi-quasi Hamiltonian systems.

Abstract: As is well known, a quasi-bi-Hamiltonian system has a standard Hamiltonian representation with respect to one Poisson structure and a quasi-Hamiltonian representation (i.e. Hamiltonian up to a factor) with respect to the other. Bi-quasi Hamiltonian systems are introduced as systems which have a double quasi-Hamiltonian representation and satisfy a further compatibility condition which has to do with the compatibility of Poisson structures on an extended space. It will be shown how such systems also give rise to an algorithm for constructing integrals in involution, leading to complete integrability under certain circum-stances. A couple of examples will be discussed.

Alexander Shapovalov
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Title: Selected topics in classical separation of variables and applications.

Abstract:

Evgeny Sklyanin
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Title: Q-operator and separation of variables"

Abstract: Q-operator can be considered as a quantization of classical Backlund transformation. It shares eigenfunctions with the commuting Hamiltonians defining a quantum integrable system and it eigenvalues satisfy a simple differential or difference equation in an auxiliary parameter. We show that a composition of several Q-operators produces a universal construction of separation of variables. The general scheme is illustrated on the example of the Calogero-Sutherland system.

Giorgio Tondo

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Title: Generalized Lenard chains and separation of variables.

Abstract: It is known that integrability of soliton equations follows from the existence of Lenard chains of symmetries,constructed by means of a Nijenhuis (or hereditary) operator.Until recently,such an operator was thought to play no role in the theory of classical (finite-dimensional)integrable systems.In this talk,we will discuss the prominent role of Nijenhuis operators in the problem of separation of variables (SoV) for the classical Hamilton-Jacobi (HJ)equation.We will show that the existence of suitable generalized Lenard chains,for a given finite-dimensional Hamiltonian system,ensures that the corresponding HJ equation can be solved by SoV in a set of special coordinates of the phase space,naturally de fined by a Nijenhuis operator.Finally,we will discuss some examples of generalized Lenard chains and,in particular,the so-called quasi-bi-Hamiltonian systems.

Claes Waksjö

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Title: Separation of quasi-potential Newton equations and cofactor-elliptic coordinates.

Abstract:

Pavel Winternitz

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Title: Integrability and superintegrability without separation of variables.

Abstract: We consider finite dimensional integrable and superintegrable systems with velocity dependent potentials and second order integrals of motion, or velocity independent potentials and higher order integrals. In both cases new phenomena occur. First of all, the classical and quantum integrable Hamiltonians do not coincide. Secondly, integrability no longer implies separability